

**ANÁLISE DE UM SISTEMA SOLAR TÉRMICO DE  
CIRCUITO FECHADO PELO MÉTODO “ $\bar{\Phi}$ /F-CHART” :  
EFEITO DE DIFERENTES LOCALIDADES  
BRASILEIRAS**

**ANALYSIS OF A CLOSED LOOP SOLAR ENERGY  
SYSTEM USING THE  $\bar{\Phi}$ -F CHART METHOD: EFFECT  
OF DIFFERENT BRAZILIAN LOCATIONS**

**Alexandre M. S. Costa\***

**RESUMO:**

O projeto preliminar ou sistemas solares térmicos pequenos requerem métodos não onerosos para predição do desempenho energético de longa duração. O método “ $\bar{\Phi}$ /f chart” combina duas abordagens. Usabilidade pode ser definida como a fração da radiação solar incidente que pode ser convertida em calor aproveitável. Se somente radiação acima de uma intensidade é aproveitável, podemos definir uma estatística de radiação solar, denominada de usabilidade, como a fração da radiação total recebida com uma intensidade acima da aproveitável. Multiplicando a radiação solar média para um dado período de tempo por esta fração obtem-se a energia total utilizável. Por sua vez, o método “f-chart” fornece uma metodologia para estimar a fração f da carga térmica de aquecimento mensal que pode ser fornecida por um sistema solar ativo. Neste trabalho, utilizamos o método “ $\bar{\Phi}$ /f-chart” para investigar o desempenho energético anual de um sistema solar térmico de água quente sob o efeito de diferentes condições climáticas, para diferentes localidades brasileiras. O efeito das perdas de calor no tanque de armazenamento de água quente é também investigado.

**Palavras chaves:** Sistemas solares. Projeto de engenharia. Energias renováveis

---

\*Universidade Estadual de Maringá, e-mail: amscosta@uem.br

**ABSTRACT:**

Preliminary designs of small solar thermal systems require inexpensive methods for predicting long-term energy performance. The  $\bar{\Phi}$ - $f$ -chart method combines two approaches. Usability can be defined as the amount of incident solar radiation that can be converted in usable heat. If only radiation above a specific intensity is usable, then we can define a statistic of solar radiation, called “usability,” as the fraction of the total radiation that is received at an intensity higher than the usable level. Multiplying the average solar radiation for a given period by this fraction provides the figure for total usable energy. In turn, the  $f$ -chart method provides a procedure for estimating the fraction,  $f$ , of the total monthly heating load that can be supplied by an active solar energy system. In this study, we use the  $\bar{\Phi}$ - $f$ -chart method to investigate the annual energy performance of a hot-water solar thermal system under different climatic conditions, for different Brazilian locations. The effect of heat losses in the hot water storage tank is also investigated.

**Keywords:** Solar thermal systems. Engineering design. Renewable energy.

**INTRODUCTION**

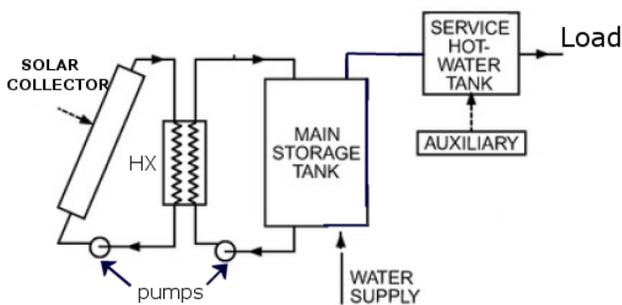
The  $\bar{\Phi}$ - $f$  chart method was proposed by Klein and Beckman (1979) to estimate the long-term performance of solar heating systems and for designing process heat and power systems. According to the ASHRAE (2011) it is one of the main methods to produce estimates of annual useful outputs of solar thermal processes. The  $\bar{\Phi}$ - $f$  chart method combines two design approaches and can be used whenever the average operating temperature of the solar collector is known.

This method basically requires the monthly averaged daily radiation incident on the horizontal surface, which is available in the databases of national weather services of many countries. For many systems with thermal storage, the analysis is complex because the collector temperature fluctuates with storage temperature, which in turn depends on collector output during previous periods. In order to overcome this difficulty, the  $\bar{\Phi}$ - $f$  chart method is used. The method calculates  $f$ , the fraction of monthly heat load supplied by solar energy, as a function of two dimensionless parameters: the ratio of collector losses to heating loads and absorbed radiation to heating loads.

Figure (1) shows a schematized solar system that can be analyzed by the  $\bar{\Phi}$ - $f$  chart method. The solar energy closed loop is active, i.e. the pumps are on, whenever the temperature of the main storage

tank is above  $T_{\min}$ , the minimum temperature usable by the load. The backup or auxiliary is in parallel with the solar system and supplies the entire load whenever storage falls below  $T_{\min}$ . A heat exchanger (HX) is used for transferring energy between the solar collector fluid loop and the main storage tank loop.

**Figure 1.** Closed loop hot water solar energy system



Transient simulations are rapidly replacing design methods such as the  $\bar{\Phi}$ - $f$  chart method as a result of the increasing computational speed of computers. However, design methods are still much faster and are therefore useful for early general survey studies, where complete simulations are too expensive. An additional advantage of using the  $\bar{\Phi}$ - $f$  chart method is because of its simple analytical formulation, as opposed to an annual full-time transient simulation performed in codes such as TRNSYS (2005).

**BASIC EQUATIONS**

All equations will be provided in a stepwise line:

$$\delta = 23.45 \sin\left(360 \frac{284+n}{365}\right) \quad (1)$$

$$\omega_s = \cos^{-1}(-\tan(\phi) \tan(\delta)) \quad (2)$$

In Eqs. (1) and (2),  $\delta$  is the sun declination in degrees from Cooper (1969);  $n$  is the month central day ( $1 \leq n \leq 365$ );  $\omega_s$  is the sunset hour angle in degrees;  $\phi$  is the location latitude in degrees.

$$\bar{H}_o = \frac{24 \times 3600 G_{sc}}{\pi} \left(1 + 0.033 \cos\left[\frac{360 n}{365}\right]\right) \times \left(\cos[\phi] \cos[\delta] \sin[\omega_s] + \frac{\pi \omega_s}{180} \sin[\phi] \sin[\delta]\right) \quad (3)$$

$$\bar{K}_T = \frac{\bar{H}}{\bar{H}_o} \quad (4)$$

In Eqs. (3) and (4),  $\bar{H}_o$  is the daily extraterrestrial solar radiation on a horizontal surface for the month central day in  $J/m^2$ ;  $G_{sc}$  is the solar constant

(=  $1367 \text{ W/m}^2$ );  $\bar{H}$  is the monthly averaged daily radiation on a horizontal surface in  $J/m^2$ , which is attainable from weather station data; and  $\bar{K}_T$  is the monthly averaged clearness index.

$$\bar{R}_b = \frac{\cos(\phi + \beta) \cos(\delta) \sin(\omega_s) + (\pi/180) \omega_s \sin(\phi + \beta) \sin(\delta)}{\cos(\phi) \cos(\delta) \sin(\omega_s) + (\pi/180) \omega_s \sin(\phi) \sin(\delta)} \quad (5)$$

$$\omega_s = \min\left[\cos^{-1}(-\tan(\phi) \tan(\delta)), \cos^{-1}(-\tan(\phi + \beta) \tan(\delta))\right] \quad (6)$$

In Eqs. (5) and (6), in accordance with Liu and Jordan (1962) and Klein (1977),  $\bar{R}_b$  is the ratio of the average daily beam radiation on the tilted surface to that on a horizontal surface for each month;  $\omega_s$  is the sunset hour angle for the tilted

surface for the central day of the month;  $\beta$  is the solar collector tilt angle measured from horizontal. These previous equations are valid for collector azimuth angles of 180 degrees, i.e. the collector faces the north.

$$\frac{\bar{H}_d}{\bar{H}} = 1.391 - 3.560 \bar{K}_T + 4.189 \bar{K}_T^2 - 2.137 \bar{K}_T^3 \quad \omega_s \leq 81.4^\circ \quad 0.3 \leq \bar{K}_T \leq 0.8 \quad (7)$$

$$\frac{\bar{H}_d}{\bar{H}} = 1.311 - 3.022 \bar{K}_T + 3.427 \bar{K}_T^2 - 1.821 \bar{K}_T^3 \quad \omega_s > 81.4^\circ \quad 0.3 \leq \bar{K}_T \leq 0.8 \quad (8)$$

$$\bar{R} = \left(1 - \frac{\bar{H}_d}{\bar{H}}\right) \bar{R}_b + \frac{\bar{H}_d}{\bar{H}} \left(\frac{1 + \cos(\beta)}{2}\right) + \rho_g \left(\frac{1 - \cos(\beta)}{2}\right) \quad (9)$$

In Eqs. (7) and (8), in accordance with Erbs et al. (1982),  $\bar{H}_d/\bar{H}$  is the ratio of monthly averaged daily diffuse solar radiation to the monthly averaged daily radiation. In Eq. ,  $\bar{R}$  is the ratio of monthly averaged daily total solar radiation on the collector to the monthly averaged daily radiation. Based on the Liu Jordan (1962) isotropic sky model and extended by Klein (1977),  $\rho_g$  is the ground solar reflectivity.

$$r_{t,n} = \frac{I}{H} = \frac{\pi}{24} (a^* + b^*) \frac{1 - \cos(\omega_s)}{\sin(\omega_s) - \frac{\pi\omega_s}{180} \cos(\omega_s)} \quad (10)$$

$$a^* = 0.409 + 0.5016 \sin(\omega_s - 60) \quad (11)$$

$$b^* = 0.6609 - 0.4767 \sin(\omega_s - 60) \quad (12)$$

$$\omega_s \leq 81.4^\circ: \quad \frac{H_d}{H} = \begin{cases} 1.0 - 0.272 K_T + 2.4495 K_T^2 - 11.95 K_T^3 + 9.3879 K_T^4 & K_T < 0.715 \\ 0.143 & K_T \geq 0.715 \end{cases} \quad (15)$$

$$\omega_s > 81.4^\circ$$

$$\omega_s > 81.4^\circ: \quad \frac{H_d}{H} = \begin{cases} 1.0 + 0.283 K_T - 2.5557 K_T^2 + 0.8448 K_T^3 & K_T < 0.722 \\ 0.175 & K_T \geq 0.722 \end{cases} \quad (16)$$

In Eqs. (15) and (16), in accordance with Erbs et al (1982),  $H_d/H$  is the ratio of daily diffuse solar radiation to the daily radiation for the month central day.  $K_T$  is the ratio of total daily radiation to the extraterrestrial radiation for the month central day.

$$r_{d,n} = \frac{\pi}{24} \frac{1 - \cos(\omega_s)}{\sin(\omega_s) - \frac{\pi\omega_s}{180} \cos(\omega_s)} \quad (13)$$

In Eqs. (10) to (12), in accordance with Collares-Pereira and Rabl (1979),  $r_{t,n}$  is the ratio of  $I$ , the total solar for the noon hour, to  $H$ , the total solar daily radiation for the month central day. In Eq. , in accordance with Liu and Jordan (1960),  $r_{d,n}$  is the ratio of diffuse solar radiation for the noon hour to the total solar daily radiation for the month central day.

$$R_{b,n} = \frac{\cos|-\phi + \delta - \beta|}{\cos|-\phi + \delta|} \quad (14)$$

In Eq.(14),  $R_{b,n}$  is the ratio of beam radiation on the collector surface to that on a horizontal surface for the noon hour.

In Eq. (17),  $R_n$  is the ratio of  $I_T$ , the noon hour total solar radiation on the collector, to  $I$ , the noon hour total solar radiation on a horizontal surface. The last equation is based on the Liu and Jordan (1963) isotropic sky model. The equation that follows is for the  $\Phi$ -f chart method.

$$R_n = \left( \frac{I_T}{I} \right)_n = \left( 1 - \frac{r_{d,n} H_d}{r_{t,n} H} \right) R_{b,n} + \left( \frac{r_{d,n} H_d}{r_{t,n} H} \right) \left( \frac{1 + \cos(\beta)}{2} \right) + \rho_g \left( \frac{1 - \cos(\beta)}{2} \right) \quad (17)$$

$$\bar{X}_{c,min} = F_R U_L \cdot (T_{min} - \bar{T}_a) \cdot \frac{3600}{\left( F_R (\tau\alpha)_n \cdot \left( \frac{\bar{\tau\alpha}}{(\tau\alpha)_n} \right) \cdot r_{t,n} \cdot R_n \cdot \bar{H} \cdot 1 \times 10^6 \right)} \quad (18)$$

In Eq. (18),  $\bar{X}_{c,\min}$  is the minimum monthly averaged ratio of absorbed solar radiation to the heating loads.  $F_R$  is the combined solar collector heat removal factor and heat exchanger efficiency.  $U_L$  is the overall collector heat transfer coefficient.  $T_{\min}$  is the minimum temperature usable by the load.  $T_a$  is the monthly averaged ambient temperature that is obtained from meteorological data.  $(\tau\alpha)_n$  is the product transmissivity and absorptivity at normal incidence. The property is a glass enclosure and collector plate.  $\tau\alpha/(\tau\alpha)_n$  is the incidence angle modifier.

$$\bar{\Phi}_{\max} = \exp \left\{ \left[ a + b \left( \frac{R_n}{R} \right) \right] \left[ \bar{X}_{c,\min} + c \bar{X}_{c,\min}^2 \right] \right\} \quad (19)$$

$$a = 2.943 - 9.271 \bar{K}_T + 4.031 \bar{K}_T^2 \quad (20)$$

$$b = -4.345 + 8.853 \bar{K}_T - 3.602 \bar{K}_T^2 \quad (21)$$

$$c = -0.170 - 0.306 \bar{K}_T + 2.936 \bar{K}_T^2 \quad (22)$$

$$f = Y \bar{\Phi}_{\max} - 0.015 \left( \exp(3.85 \cdot f) - 1 \right) \left( 1 - \exp(-0.15 X') \right) R_s^{0.76} \quad (25)$$

In Eq.(25),  $f$  is the fraction of the monthly heating load (for space heating and hot water) supplied by solar energy.  $R_s$  is the ratio of the standard storage heat capacity per unit of collector area ( $= 350 \text{ kJ m}^{-2} \text{ C}^{-1}$ ) to the actual storage capacity per unit of collector area. In the next two equations the storage tank heat losses are considered.

$$\dot{Q}_{st} = (UA)_s (T_s - T_a') \quad (26)$$

$$Q_{st} = (UA)_s (\bar{T}_s - T_a') \Delta t \quad (27)$$

In Eq. (19),  $\bar{\Phi}_{\max}$  is the daily maximum utilizability as sum for a month, over all hours and all days, of the radiation on a tilted surface that is above a critical level divided by the monthly radiation.

$$\bar{\Phi}_{\max} Y = \bar{\Phi}_{\max} \frac{A_c F_R (\tau\alpha)_n \cdot \left( \frac{\tau\alpha}{(\tau\alpha)_n} \right) \bar{H} \bar{R} N}{L} \quad (23)$$

In Eq. (23), the product  $\bar{\Phi}_{\max} Y$  corresponds to the  $\bar{\Phi}$ - $f$  chart method ordinate, where  $N$  is the number of days of the month;  $A_c$  is the collector area, and  $L$  is the monthly total heating load for space heating and hot water in  $J$ .

$$X' = \frac{A_c F_R U_L (100) \Delta t}{L} \quad (24)$$

In Eq. (24), the variable  $X'$  corresponds to the  $\bar{\Phi}$ - $f$  chart method abscissa and  $\Delta t$  is the total number of seconds in the month.

In Eqs. (26) and (27),  $\dot{Q}_{st}$  is the instantaneous heat transfer loss in  $W$ ;  $(UA)_s$  is the product corresponding to the global heat transfer coefficient and heat transfer area for the storage tank.  $T_s$  is the instantaneous tank temperature and average ambient temperature for the storage tank.  $Q_{st}$  is the monthly heat transfer storage loss, and  $\bar{T}_s$  is the average tank temperature. In the previous equations, the environment temperature where the storage tank is placed,  $T_a'$ , is considered constant.

$$f_{TL} = \frac{L_s + Q_{st}}{L + Q_{st}} \quad (28)$$

$$f = f_{TL} \left( 1 + \frac{Q_{st}}{L} \right) - \frac{Q_{st}}{L} \quad (29)$$

In Eqs. (28) and (29),  $f_{TL}$  is the fraction of the monthly load supplied by solar energy (including tank losses) and  $L_s$  is the solar energy supplied to the load. In obtaining Eq. (29),  $f$  as  $L_s/L$  is introduced in Eq. (28).

$$\bar{\Phi} = \frac{f_{TL}}{Y} \quad (30)$$

In Eq. (30),  $\bar{\Phi}$  represents the average daily utilizability.

The procedure for considering the tank thermal loss is iterative. The steps, based on Duffie and Beckman (2006), are depicted next.

- I. An estimate is first made of the monthly average tank temperature,  $\bar{T}_s$
- II.  $Q_{st}$  is calculated from Eq. (27).
- III. The ordinate given by Eq. (23) is corrected considering the total load as  $L + Q_{st}$ , i.e. multiplying the right hand side of Eq (23) by  $L/(L + Q_{st})$ .
- IV. The abscissa given by Eq. (24) is corrected, considering the total load as  $L + Q_{st}$ , i.e. multiplying the right hand side of Eq (24) by  $L/(L + Q_{st})$ .
- V.  $f_{TL}$  is obtained from Eq. (25)
- VI.  $\bar{\Phi}$  is obtained from Eq. (30).
- VII.  $X_c$  variable is obtained by Eq. (19), dropping all the subscripts, i.e. using  $\bar{\Phi} = \bar{\Phi}_{max}$  and  $X_c = \bar{X}_{c,min}$ .
- VIII.  $\bar{T}_i$  is obtained from Eq. (18), using  $\bar{T}_i = T_{min}$  and  $X_c = \bar{X}_{c,min}$ .
- IX. The new temperature guess is the average of  $T_{min}$  and  $\bar{T}_i$ .

- X. If the temperature convergence is satisfactory go to step XI. If not, return to step II.
- XI. The final value of  $Q_{st}$  is used in Eq. (29) obtaining  $f$ , the fraction of load supplied by solar.

## RESULTS AND DISCUSSION

The calculation procedure described in the previous section was implemented for a typical solar system, such as the one depicted in Figure 1, using the thermodynamics software EES (Klein, 1993), for different Brazilian cities. Some properties of the solar equipment were fixed, and they are described in Table 1. The load rate  $\dot{L}$  is required during a 12-hour period each day; the storage tank capacity is 4180 liters of water. In this analysis,  $\tau\alpha/(\tau\alpha)_n$  is considered a property of the solar collector, given as the vector [0.94,0.94,0.93,0.93,0.91,0.90,0.91,0.92,0.93,0.94,0.93,0.94]. The analysis was performed for six Brazilian cities, namely: Curitiba, São Paulo, Rio de Janeiro, Recife, Salvador and Brasilia. The corresponding latitudes for the previous cities are given by the vector [-25.52,-23.62,-22.82,-8.07,-12.90,-15.87].

**Table 1.** Baseline parameters.

Parameter	Value
$\beta$	40°
$\rho_g$	0.2
$T_{min}$	60 C
$F_R (\tau\alpha)_n$	0.72
$F_R U_L$	2.63 W/(m <sup>2</sup> C)
$A_c$	50 m <sup>2</sup>
$\dot{L}$	12 kW

The climatic data for performing the calculations were:  $\bar{H}$ , the monthly averaged daily radiation on a horizontal surface, and  $\bar{T}_a$ , the monthly averaged ambient temperature. Those values, obtained from the DOE website (2011), are depicted in Figs. 2 and 3.

Figure 2. The monthly averaged daily radiation on a horizontal surface by location

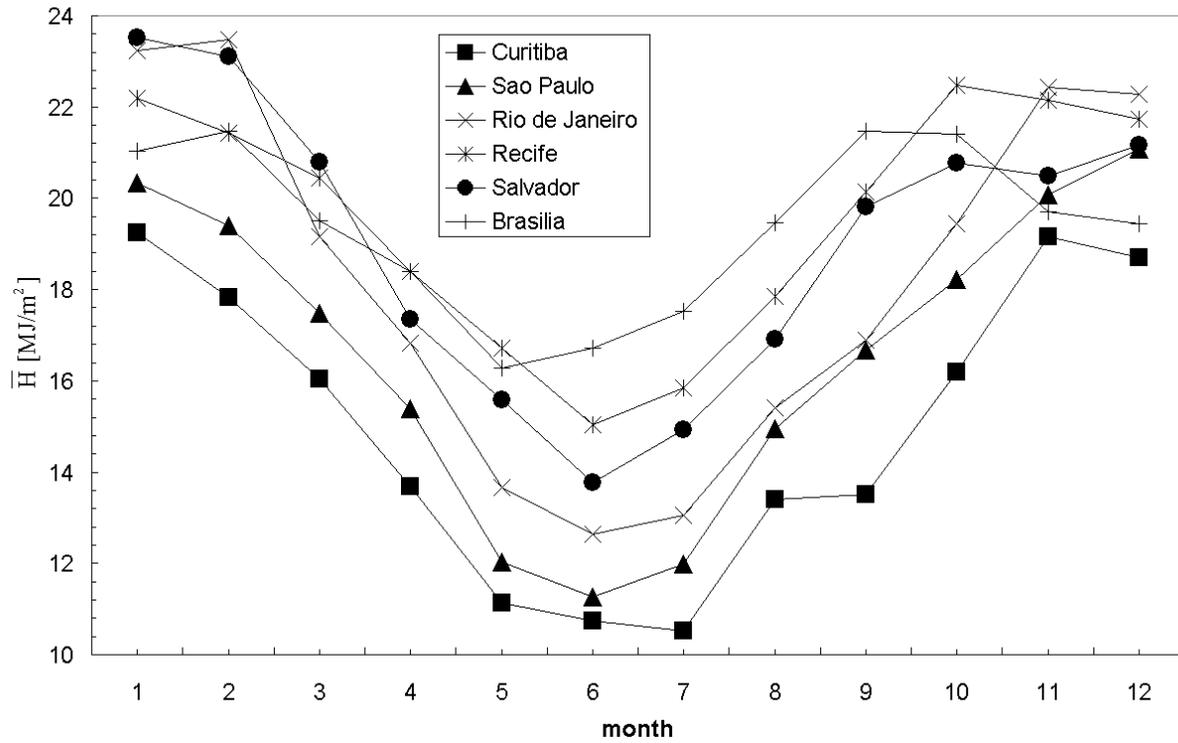


Figure 3. The monthly averaged daily temperature by location

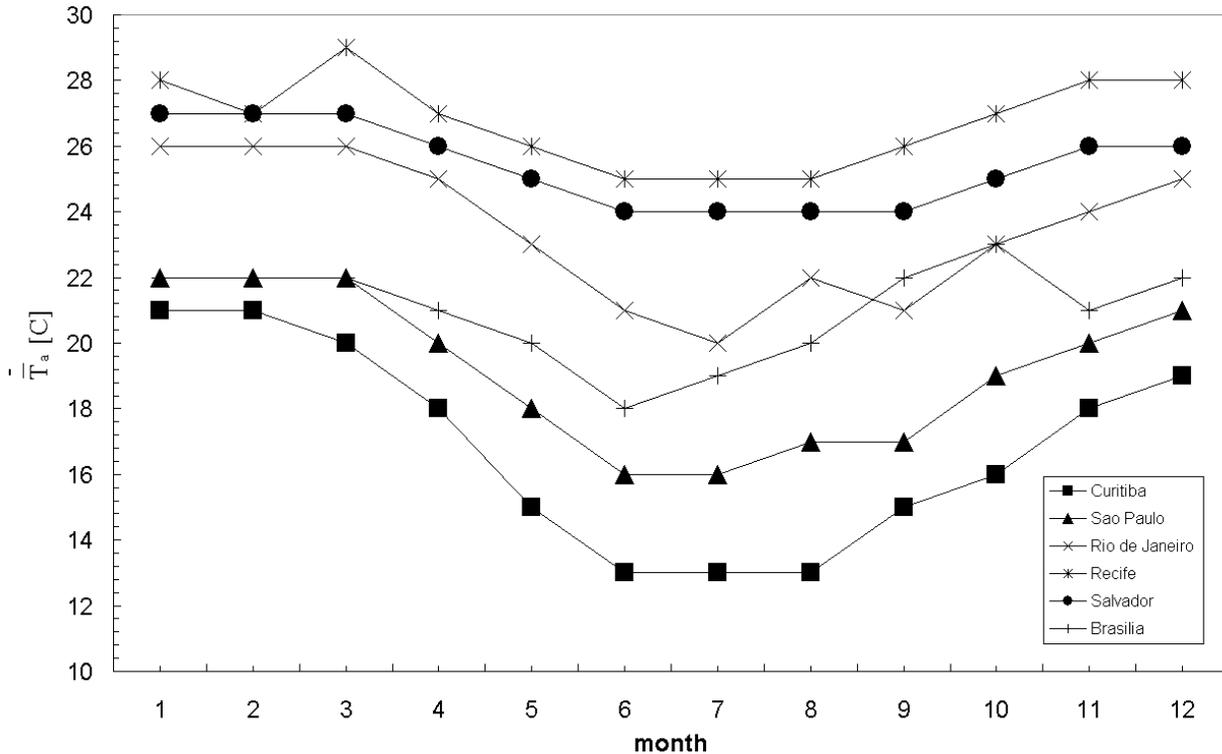


Figure 4. The monthly solar load fraction by location

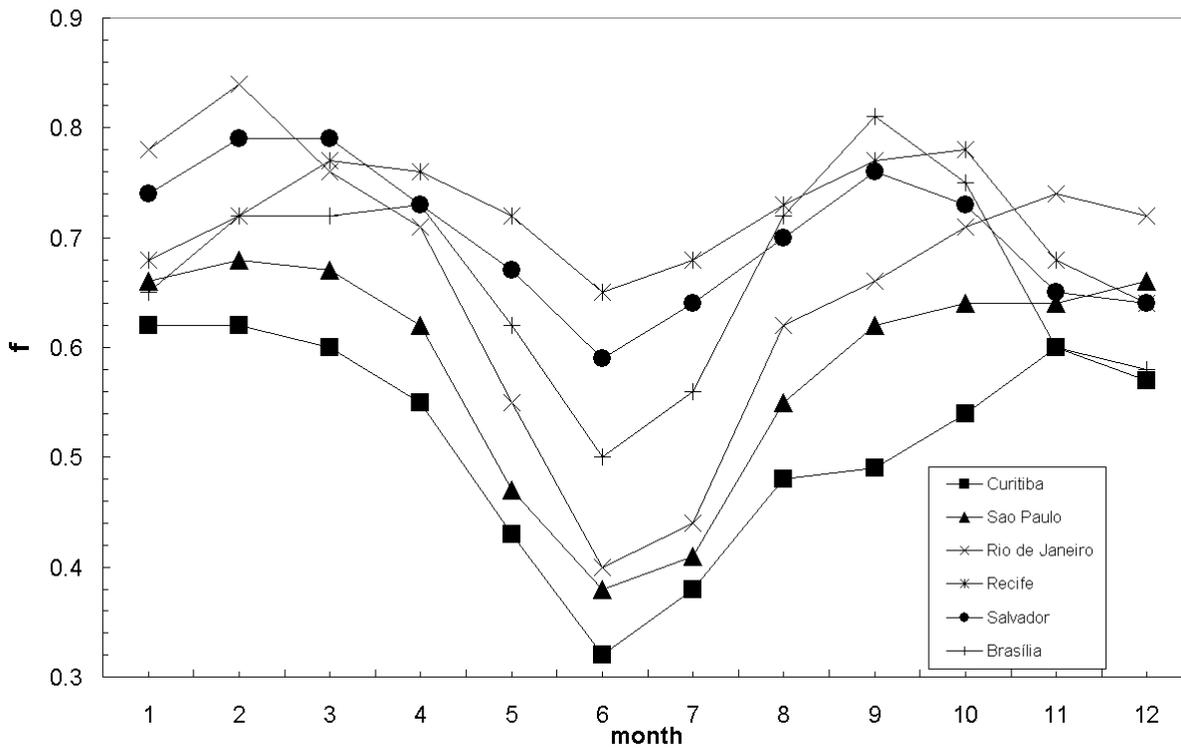


Figure 4 presents the monthly solar load fraction for all the seven Brazilian cities; no storage tank heat losses are considered. The annual behavior of the fraction of the load from solar presents similar trends to the monthly average temperature and global horizontal radiation, with a maximum in the summer and minimum in the winter months. Between the fall and the spring, the *f* value increases with the latitude of the cities. During the summer, the pattern was broken, with Rio de Janeiro presenting the highest fraction, followed by Salvador and Recife. For the winter months, Curitiba and São Paulo exhibited the lowest values of solar contribution. Again, this pattern agrees with the global radiation behavior in the summer months. The ranking of the cities by percentage variation in the solar contribution ( $f_{max} - f_{min}$ )/ $f_{max}$  was: Recife (17 %), Salvador (25%), Brasília (38%), São Paulo (44%), Curitiba (48%) and Rio de Janeiro (52%).

When considering the effect of the storage tank heat losses, the products  $UA_s$  and  $T_a$  were  $5.9 \text{ W m}^{-2} \text{ C}^{-1}$  and  $20 \text{ C}$ , respectively. Table 2 shows the ranking of annual solar contribution to the load: Curitiba (52%), São Paulo (58%), Rio de Janeiro and Brasília (66%), Salvador (70%) and Recife (71%) –

no tank losses. The ranking is unchanged with tank losses. Furthermore, the calculation indicates an average reduction of 4% in the solar contribution when tank losses are considered.

Table 2. *f* – Annual solar load fraction

	No loss	Loss
Curitiba	0.52	0.48
São Paulo	0.58	0.55
Rio de Janeiro	0.66	0.63
Recife	0.71	0.69
Salvador	0.70	0.67
Brasília	0.66	0.63

### CONCLUSION

The calculation procedure of the  $\bar{\Phi}$ -*f* chart method was carried out for a typical solar system for different Brazilian cities. The solar contribution was greater in the summer, and less in the winter. In general, between the fall and the spring, the contribution increased with the latitudes of the cities. The pattern was broken during the summer, with Rio de Janeiro displaying the highest solar contribution. For the cases investigated, heat losses from the storage tank decreased the solar contribution

by 4%. The annual solar contribution ranking for the cases studied was Recife (the highest), Salvador, Brasília and Rio de Janeiro, São Paulo and Curitiba (the lowest).

## REFERENCES

ASHRAE (American Society of Heating, Refrigerating and Air-Conditioning Engineers) 2011. ASHRAE Handbook. **HVAC applications**, Atlanta : USA, 2011. Chapter 35.

COLLARES-PEREIRA, M., RABL, A. The average distribution of solar radiation - correlations between diffuse and hemispherical and between daily and hourly insolation values. **Solar Energy**, v.22, p. 155-164, 1979.

COOPER, P. I. The absorption of solar radiation in solar stills. **Solar Energy**, v.12, n. 3, p. 332-340, 1969.

DOE. Energy plus energy simulation software: weather data, available at: <[http://apps1.eere.energy.gov/buildings/energyplus/cfm/weather\\_data.cfm](http://apps1.eere.energy.gov/buildings/energyplus/cfm/weather_data.cfm)> Accessed 01/06/2011

DUFFIE, J. A., BECKMAN, W. A. **Solar Engineering of Thermal Processes**, Wiley; 3rd edition, 928p, 2006.

ERBS, D. G., KLEIN, S. A., DUFFIE, J. A. Estimation of the diffuse radiation fraction for hourly, daily, and monthly average global radiation, **Solar Energy**, 28, p. 293-302, 1982.

KLEIN, S. A. Calculation of monthly averaged insolation on tilted surfaces, **Solar Energy**, 19, p. 325-329, 1977.

\_\_\_\_\_. Calculation of Flat-Plate collector utilizability. **Solar Energy**, 21, p. 393-402, 1978.

\_\_\_\_\_. Development and integration of an equation-solving program for engineering thermodynamics courses, **Computer Applications in Engineering Education**, v. 1, n. 3, p. 265- 275, 1993.

\_\_\_\_\_; BECKMAN, W. A. A generalized design method for closed-loop solar energy systems. **Solar Energy**, 22, p. 269-282, 1979.

LIU, B. Y. H., JORDAN, R. C., The interrelationship and characteristic distribution of direct, diffuse and total solar radiation. **Solar Energy**, v.4, p.3, p. 1-19, 1960.

\_\_\_\_\_. JORDAN, R. C. Daily insolation on surfaces tilted toward the equator. **ASHRAE Journal**, v.3, n.10, p. 53, 1962.

LIU, B. Y. H.; JORDAN, R. C. The long term average performance of flat-plate solar energy collectors. **Solar Energy**, 7, p. 53-74, 1963.

TRNSYS 16. **A TRaNsient system simulation program**. University of Wisconsin Solar Energy Laboratory, Madison, Wisconsin, USA, 2005.